Note

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Introduction to Mechanics 力学简介：

Mechanics 力学 is a branch of physics that studies the relationships between **energy**, **force**, and the **motion** or **deformation** of objects (solids, liquids, and gases). It focuses on understanding how forces affect the equilibrium, motion, and interactions of matter under various conditions. Classical mechanics, the foundation of this field, is divided into three main categories:

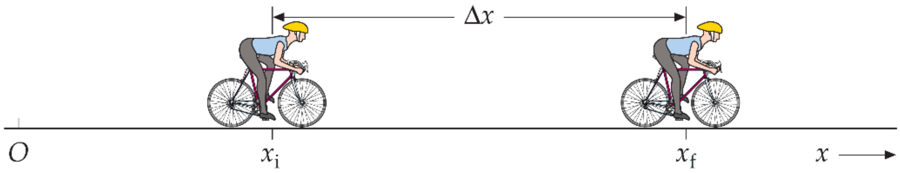
- **Kinematics 运动学**: Describes motion without considering its causes (e.g., velocity, acceleration).
- **Statics 静力学**: Analyzes forces in systems at rest or in equilibrium (e.g., bridges, buildings).
- **Dynamics 动力学**: Explores the connection between forces and motion (e.g., Newton's laws, planetary orbits).

The following list is some mechanics related courses in Tsinghua University:

- Classical Mechanics 理论力学
- Engineer Mechanics 工程力学
  - Mechanics of Materials 材料力学, Elasticity 弹性力学, Plasticity 塑性力学, Structural Mechanics 结构力学
- Fluid Mechanics 流体力学
  - Advanced Fluid Mechanics 高等流体力学, hydraulics 水力学，Viscous Fluid Flow 粘性流体力学，Aerodynamics 空气动力学，Electromagnetic Fluid Dynamics 电磁流体力学, Introduction to Turbulence 湍流概论, Aeroacoustics and Aerodynamic Noise 航空声学与气动噪声
- Solid Mechanics 固体力学
  - Advanced Solid Mechanics 高等固体力学, Fracture Mechanics 断裂力学
- Computational and Experimental Mechanics 计算力学和实验力学
  - Finite Element Method 有限元法, Computational Fluid Mechanic 计算流体力学, Computational Solid Mechanics 计算固体力学, Tensor Analysis 张量分析, Advanced Computational Fluid Mechanics 高等计算流体力学，Experimental Stress Analysis 实验应力分析
- Others
  - Cell Biomechanics and Clinical Applications 细胞生物力学与临床应用, Aerospace Materials and Structures 飞行器材料与结构, Composite Structure Design 复合材料结构设计, Flexible Electronics Technology 柔性电子技术, Physical Mechanics 物理力学, Advanced Biomechanics 高等生物力学

2-1 POSITION AND DISPLACEMENT位置和位移

If we only examine the motion of an object without considering its shape, deformation, or internal motion, we can simplify the object as a point with mass, called a **particle质点**. To describe the motion of a particle, we need to describe its **position位置** in a **coordinate system** and how its position changes as it moves. This coordinate system is called a **reference frame参照系**. For one-dimensional motion, we often choose the x axis as the line along which the motion takes place.



$\Delta x = x_f - x_i$

Figure 2-1

Figure 2-1 shows a student on a bicycle at position x_i at time t_i. At a later time, t_f The student is at position x_f. The **displacement位移** is *the change in position of the particle*. The **distance距离/路程** traveled by a particle is the length of the path a particle takes from its initial position to its final position.

$$\Delta\vec{x} = \vec{x}_f - \vec{x}_i$$

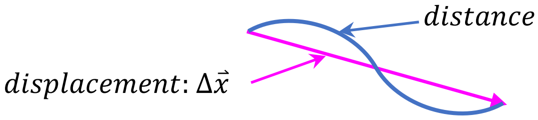


Figure 2-2 Distance and Displacement

The SI unit of position, displacement and distance are meter (m).

Example 2-1 You are playing a game of catch with a dog. The dog is initially standing near your feet. Then he jogs 20 feet in a straight line to retrieve a stick, and carries the stick 15 feet back toward you before lying on the ground to chew on the stick.

(a) What is the net displacement of the dog?
(b) What is the total distance the dog travels?
(c) Show that the net displacement for the trip is the sum of the sequential displacements that make up the trip.

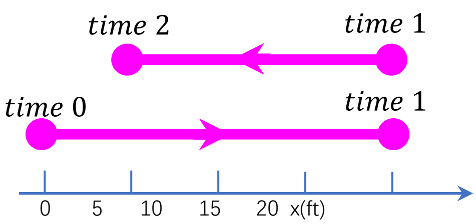


Figure 2-3

(a) Find out the position of the dog in time 0, time 1 and time 2 respectively

At time 0, the position $x_0 = 0$

At time 1, the position $x_1 = 20$

At time 2, the position $x_2 = 5$

From the definition of displacement $\Delta x = x_f - x_i$ where $x_i = x_0 = 0$, $x_f = x_2 = 5$,

$$\Delta x_{02} = x_f - x_i = 5 - 0 = 5\ (ft)$$

(b) From time 0 to time 1, the distance traveled $s_{01} = |20 - 0| = 20$

From time 1 to time 2, the distance traveled $s_{12} = |5 - 20| = 15$

The total distance travelled $s_{02} = s_{01} + s_{12} = 20 + 15 = 35\ (ft)$

(c) From time 0 to time 1, the displacement $\Delta x_{01} = x_1 - x_0 = 20 - 0 = 20$

From time 1 to time 2, the displacement $\Delta x_{12} = x_2 - x_1$=5-20=-15

The total displacement is: $\Delta x_{02} = \Delta x_{01} + \Delta x_{12} = 20 - 15 = 5\ (ft)$

Note:

(1) In a coordinate system, the position, x_f, x_i, is a **vector** to indicate where is the particle.

(2) The displacement is the change in position of the particle, $\Delta x = x_f - x_i$. It is also a **vector**.

(3) The distance traveled by a particle is the length of the path a particle takes from its initial position to its final position. It is a **scalar** quantity and is always indicated by a positive number

2-2 VELOCITY AND SPEED速度和速率

The **average speed平均速率** of a particle is the total distance traveled by the particle divided by the total time from start to finish:

$$average\ speed = \frac{total\ distance}{total\ time} = \frac{s}{\Delta t}$$

Because the total distance and total time are both scalers, the average speed is a scalar (always positive).

The **average velocity平均速度**, $\bar{v}$, of a particle is defined as the ratio of the displacement Δx to the time interval Δt:

$$v_{ave} = \bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

The SI unit of speed/velocity are meters per second (m/s).

Example 2-2 The dog that you were playing catch with in Example 2-1 jogged 20.0 ft away from you in 1.0 s to retrieve the stick and ambled back 15.0 ft in 1.5 s (Figure 2-4). Calculate

(a) the dog's average speed, and

(b) the dog's average velocity for the total trip.

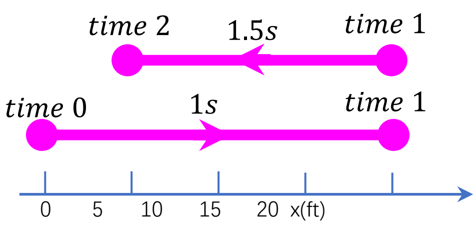


Figure 2-4

(a) Total distance traveled $s_{02} = s_{01} + s_{02} = 20 + 15 = 35\ (ft)$

Total time $t_{02} = 1 + 1.5 = 2.5\ (s)$

Average Speed $= \frac{s_{02}}{t_{02}} = \frac{35}{2.5} = 14\ (ft/s)$

(b) Net displacement $\Delta x_{02} = \Delta x_{01} + \Delta x_{12} = 20 - 15 = 5\ (ft)$

Total time $t_{02} = 1 + 1.5 = 2.5\ (s)$

Average velocity $v_{ave} = \bar{v} = \frac{\Delta x_{02}}{t_{02}} = \frac{5}{2.5} = 2\ (ft/s)$

The average velocity for the interval between t_1 and t_2 is the slope of the straight line connecting the points (t_1, x_1) and (t_2, x_2) on an x versus t graph. This is the **geometric interpretation of average velocity速度的几何学意义**.

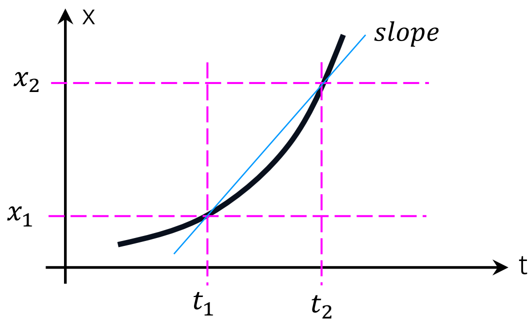


Figure 2-4

We define the **instantaneous velocity 瞬时速度**, $v_x(t)$, is the sloop of the tangent line at time t. It is also the limit of the ratio $\Delta x/\Delta t$ as Δt approaches zero (derivative).

$$v_x(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

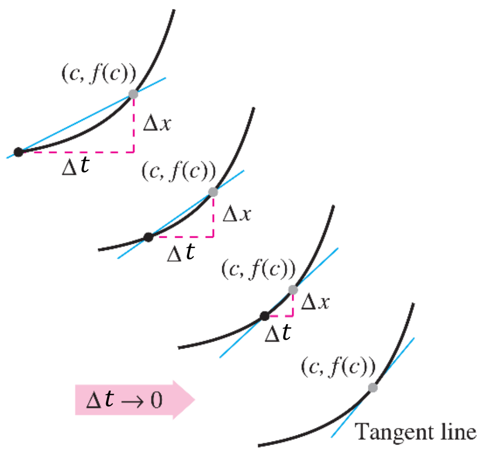


Figure 2-5

The instantaneous velocity is a vector, and the magnitude of the instantaneous velocity is the **instantaneous speed瞬时速率**. Throughout the rest of the text, we shall use "velocity" in place of "instantaneous velocity" and "speed" in place of "instantaneous speed".

Example 2-3 The position of a stone dropped from a cliff is described approximately by $x = 5t^2$, where x is in meters and t is in seconds. The $+x$ direction is downwards and the origin is at the top of the cliff. Find the velocity of the stone during its fall as a function of time t.

Solution: Definition of the velocity: $v_x(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = \frac{d}{dt}(5t^2) = 10t \;\; (m/s)$

Calculate the velocity by definition of limit (Review the calculation of limits in calculus)

$$v_x(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \lim_{\Delta t \to 0} \frac{x(t + \Delta t) - x(t)}{\Delta t}$$

Where

$$\begin{cases} x(t) = 5t^2 \\ x(t + \Delta t) = 5(t + \Delta t)^2 = 5[t^2 + 2t\Delta t + (\Delta t)^2] \end{cases}$$

So that

$$v_x(t) = \lim_{\Delta t \to 0} \frac{x(t + \Delta t) - x(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{5[2t\Delta t + (\Delta t)^2]}{\Delta t}$$
$$= \lim_{\Delta t \to 0} (10t + 5\Delta t) = 10t$$

Note

2-3 ACCELERATION加速度

The **average acceleration 平均加速度**, $a_{avg} = \bar{a}$ for a particular time interval Δt is defined as the change in velocity Δv, divided by that time interval:

$$a_{avg} = \bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \Rightarrow \Delta v = \bar{a}\Delta t$$

Instantaneous acceleration 瞬时加速度 is the limit of the ratio $\Delta v / \Delta t$ as Δt approaches zero. On a plot of velocity versus time, the instantaneous acceleration at time t is the slope of the line tangent to the curve at that time:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = slope\ of\ the\ line\ tangent\ to\ the\ v - t\ curve$$

Thus, instantaneous acceleration is the derivative of velocity v with respect to time, dv/dt.

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

From now on we will use the word acceleration to mean "Instantaneous acceleration".

Summary of physical quantities in kinematics:

vector	scaler
Position x displacement $\Delta x = x_f - x_i$ velocity $v = dx/dt$ acceleration $a = dv/dt = d^2x/dt^2$	distance s speed

Example 2-4 The domestic electric car Xiaomi Su7 announced to have an acceleration time of 2 seconds from 0 to 100 kilometers per hour. Given that the gravitational acceleration is 9.8m/s, compare the acceleration of Xiaomi Su7 with the gravitational acceleration.

$$100\ km/h = 100 \times \frac{1000\ m}{3600\ s} \approx 27.78\ \ m/s$$

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} = \frac{27.78 - 0}{\mathbf{2 - 0}} = 13.89\ \ (m/s^2)$$

$$\Rightarrow 13.89/9.8 \approx 1.42$$

The acceleration of Xiaomi Su7 is approximately 1.42 times that of gravitational acceleration.

Example 2-5 Velocity and acceleration as a function of time

The position of a particle is given by $x = Ct^3$, where C is a constant. Find the dimensions of C. In addition, find both the velocity and the acceleration as functions of time.

Solution:

$$x = Ct^3 \ \Rightarrow\ C = \frac{x}{t^3} \ \Rightarrow\ [C] = \frac{[x]}{[t^3]} = \frac{[L]}{[T^3]}$$

$$v = \frac{dx}{dt} = \frac{d}{dt}(Ct^3) = 3Ct^2$$

$$a = \frac{dv}{dt} = \frac{d}{dt}(3Ct^2) = 6Ct$$

Exercise 1: A particle moving in one dimension has a position function defined as: $x(t) = t^4 - 4t$.

(a) At what point in time does the particle change its direction along x axis?
The time the particle changing direction happens when velocity equals zero.

(b) What is its position when it changes its direction?

(c) In what direction is the body traveling when its acceleration is 3m/s^2?

Exercise 2: Known that the trajectory of a particle is a circle with radius r and its angular velocity is ω, using the definition of linear velocity, prove that $v = r\omega$.
(Tip: Circular motion can also be seen as motion in 1 dimensional, and its relationship between angular velocity ω and angular displacement θ is the same as that of velocity and displacement in 1 dimensional linear motion. $\omega = \frac{d\theta}{dt}$)

Exercise 1: A particle moving in one dimension has a position function defined as: $x(t) = t^4 - 4t$.

(a) At what point in time does the particle change its direction along x axis?

The time the particle changing direction happens when velocity equals zero.

$$v(t) = \frac{dx}{dt} = \frac{d}{dt}(t^4 - 4t) = 4t^3 - 4 = 0$$

$$\Rightarrow t = 1\ (s)$$

(b) What is its position when it changes its direction?

$$x(t = 1) = 1^4 - 4 \times 1 = -3\ (m)$$

(c) In what direction is the body traveling when its acceleration is $3\mathrm{m/s^2}$?

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}(4t^3 - 4) = 12t^2 = 3$$

$$\Rightarrow t = 0.5\ (s)$$

$$v(0.5) = 4 \cdot (0.5)^3 - 4 = -3.5\ \left(\frac{m}{s}\right)$$

Because its velocity is less than 0, its direction of motion is opposite to the positive direction of the x -axis.

Exercise 2: Known that the trajectory of a particle is a circle with radius r and its angular velocity is ω, using the definition of linear velocity, prove that $v = r\omega$. (Tip: Circular motion can also be seen as motion in 1 dimensional, and its relationship between angular velocity ω and angular displacement θ is the same as that of velocity and displacement in 1 dimensional linear motion. $\omega = \frac{d\theta}{dt}$)

$$\left.\begin{aligned} v &= \frac{dx}{dt} \\ x &= r\theta \end{aligned}\right\} \Rightarrow v = \frac{dr\theta}{dt} = r\frac{d\theta}{dt} = r\omega$$

$$\omega = \frac{d\theta}{dt}$$

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or

$$\left.\begin{aligned} \omega &= \frac{d\theta}{dt} \\ \theta &= \frac{x}{r} \end{aligned}\right\} \Rightarrow \omega = \frac{d\left(\frac{x}{r}\right)}{dt} = \frac{1}{r}\frac{d\theta}{dt} = \frac{v}{r}$$

$$\Rightarrow v = r\omega$$